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About waves in field equations



"Sein Gefieder bläht sich schwellend, Welle selbst, auf Wogen wellend, Dringt er zu dem heiligen Ort..."

J. W. von Goethe

Abstract

Field equations are the mathematical representation generally continuous assignments of events to the points of an abstract space respectively to the volume elements of an agent (e.g. a physical medium).

It involves a system of partial field equations. From waves that can pass through such an agent is no talk in them. The wave term associated with the concept of source term does not appear. Likewise, the course of the wave characterizing size of a migratory surface does not exist. It must occur at one point of the agent, spontaneous or enforced, a field size as a function of time. The wave theory of the German physicist *Karl Uller* (1872-1959), the "**Wellenkinematik**" shows a solution to the problem. We use the example of the heat conduction.

I. Field equations

Since Jean Baptiste Joseph Fourier (1786-1830), the heat conduction is represented by the following equation:

$$(G_{001}) \qquad \mathbf{W} = -\lambda \cdot grad \ T$$

 \mathbf{W} = heat flow T = abs. temperature λ = spec. thermal conductivity

Neglecting secondary effects like convection the temperature reproduction can be reproduced in a "homotrop" agent (medium) by a specified field equation:

$$(G_{002}) \qquad D \cdot c \cdot \frac{\partial T}{\partial t} = -div \mathbf{W}$$

Substituting G_001 in G_002 we get:

$$(G_003) \qquad D \cdot c \cdot \frac{\partial T}{\partial t} = div \left(\lambda \cdot grad \ T\right)$$

It is:

$$(G_{004})$$
 div $(\mathbf{A} \cdot a) = a \cdot div \mathbf{A} + \mathbf{A} \cdot grad \mathbf{A}$

Therefore:

$$(G_{005}) \qquad D \cdot c \cdot \frac{\partial T}{\partial t} = \lambda \cdot div \ grad \ T + grad \ \lambda \cdot grad \ T$$

These field equation, specified on the field size *T*, is a special case of:

$$(G_{100}) \qquad \sum_{abc} A_{abc} \cdot \frac{\partial^3 W}{\partial a \partial b \partial c} + \sum_{ab} B_{ab} \cdot \frac{\partial^2 W}{\partial a \partial b} + \sum_{c} C_a \cdot \frac{\partial W}{\partial a} + D \cdot W = 0 \qquad \left[A_{abc} = 0 \right]$$

Field equations are the mathematical representation generally continuous assignments of events to the points of an abstract space respectively to the volume elements of an agent (e.g. a physical medium). It involves a system of partial field equations which contain the state variables (field sizes) with their spatial and temporal derivatives. One can specialize such equations to a single field size. Then we obtain a specified field equation for a single field size W.

(G_101) functional sign
$$(W, W', \dot{W}) = 0$$

$$(G_{102}) \qquad W = equation(x, y, z, t)$$

From waves that can pass through such an agent is no talk in them. The wave term associated with the concept of source term does not appear. Likewise, the course of the wave characterizing size of a migratory surface does not exist. Therefore equation (G_101) and (G_102) **can not represent waves** basically!

II. Wave source

It must occur at one point of the agent, spontaneously or intentionally, a forced dependence of a field size of the time. Then there is an active wave source. There predetermined **excitation history** (**Erregungsverlauf** or **Quellungsverlauf**) at the point \mathbf{r}_0 shows (G_200).

$$(G_200)$$
 $W_0(\mathbf{r}_0,\tau)$

The locus of all field sizes W must be a closed surface because the propagation can not exclude any direction. It follows from the principle of sufficient reason¹). An enclosing of the surface must not always be available. There may also be several closed surfaces. This geometric locus is at time *t* the result of the same excitement in the source which has come into existence at the time τ .

We can write such a surface as follows:

(G_201) functional sign(**r**,
$$c_1, c_2,...$$
) = 0 $(x_0, y_0, z_0) = \mathbf{r}_0$

c1, c2,... constant surface parameters

At other times *t* this locus will represent other surfaces:

(G_202) functional sign(**r**,
$$c_1(t)$$
, $c_2(t)$,...) = 0

Or more generally $(n_1, n_2, ...$ are constant parameters of the surfaces):

(G_203) functional sign(
$$\mathbf{r}, t, n_1, n_2,...$$
) = 0

On this "flock" of "isogenic surfaces" the field size W must not be constant throughout!

The course of the excitement at another time τ will associated with another "flock" of "isogenic surfaces":

(G_204) functional sign(
$$\mathbf{r}, t, n_1(\tau), n_2(\tau),...$$
) = 0

After dissolved for τ and summarized:

$$(G_{205}) \qquad \tau = \zeta(\mathbf{r}, t)$$

By multiplying by a constant factor of the dimension 1/time follows:

$$(G_206)$$
 $\tau = \zeta(\mathbf{r}, t)$ $|v_0| \left[\frac{1}{\text{time}}\right]$

$$(G_207) \qquad v_0 \cdot \zeta(\mathbf{r}, t) = v_0 \tau$$

By introducing a new function φ , we get:

$$(G_{208}) \qquad v_0 \cdot \zeta(\mathbf{r}, t) = \varphi(\mathbf{r}, t)$$

$$(G_{209}) \qquad v_0 \cdot \zeta(\mathbf{r}, t) = \varphi(\mathbf{r}, t) = v_0 \tau$$

 $v_0 \tau$ is the excitation phase (Erregungsphase) at the time τ and identified by ϕ_0

 v_0 is the temporal excitation phase increase (zeitlicher Erregungsphasen-Anstieg)

$$(G_210) \qquad \varphi(\mathbf{r}, t) = v_0 \tau = \varphi_0$$

This implies that for a certain excitation phase $v_0 \tau$ (for a certain "flock" of "isogenic surfaces"), the function φ (**r**, *t*) has for all times *t* » τ a constant value φ_0 .

The following equation is to be set:

$$\left(\mathbf{G}_211\right) \qquad \left(\frac{d\varphi}{dt}\right)_{\tau=const.} = 0$$

A surface moves out of the source through the medium. It changes the shape, size and location continuously. On this surface applies everywhere the unchangeable function value $\varphi = \varphi_0$. This dimensionless value characterizes this surface. The surface changes in the migration!

This surface is the **migrant** (Migrant) ϕ

Currently in the origin in the source at the point \mathbf{r}_0 the migrant φ coincides for $t = \tau$ with the excitement phase v₀ τ .

$$(G_212) \qquad \varphi(\mathbf{r}_0, t) = v_0 t$$

To the various times τ (i.e. at different excitation phases) belongs at least one migrant. At time *t* many migrants φ are in space. They belong to different excitation phases.

These migrants are not "isogenic".

We have the **migrants field** at time *t*

Following relationships can be derived:

Slope of the migrants (Migranten-Gefälle)	G_300	grad $\varphi = -\mathbf{w}$
Invariance of the migrants (Migranten-Invarianz)	G_310	$\varphi(\mathbf{r}_1, t_0) = v_0 \tau$
	G_311	$\varphi(\mathbf{r}_2, t_0) = v_0 \tau$
	G_312	$\varphi(\mathbf{r}_1, t_0) = \varphi(\mathbf{r}_2, t_0)$
	G_313	grad $\varphi(\mathbf{r}_1, t_0) = grad \varphi(\mathbf{r}_2, t_0)$
	G_314	$\mathbf{w}(\mathbf{r}_1, t_0) = \mathbf{w}(\mathbf{r}_2, t_0)$
Phase difference area (Phasendiffernzfläche)	G_320	$\phi = v_0 t - \varphi(\mathbf{r}, t)$
	G_321	$\varphi(\mathbf{r}, t) \neq V_0 t$
	G_322	$\varphi(\mathbf{r}, t) = \mathbf{v}_0 t - \phi(\mathbf{r}, t)$
	G_323	$\phi(\mathbf{r}_0, t) = 0$
	G_324	$\mathbf{w} = grad \phi$
Increase of the migrants (Migranten-Anstieg)	G_330	$\dot{\varphi} \left(= \frac{\partial \varphi}{\partial t} \right)$
	G_331	$\dot{\boldsymbol{\varphi}} = \boldsymbol{V}_0 - \dot{\boldsymbol{\varphi}} \left(\mathbf{r}, t \right)$

III. Wave equations

Now we want to create a wave. We need:

1. In order to give *W* a dimension, a dimensional size

$$(G_400)$$
 $\omega = \omega(\mathbf{r},t)$

- 2. A waveform $F(\varphi)$
- 3. A mathematical expression of *W* containing the migrants

The function $\boldsymbol{\omega} = \boldsymbol{\omega}(\mathbf{r}, t)$ (G_400) is the **intensity field** (Stärkefeld) of the wave W

The wave history (Wellenverlauf) W of the same field size W is therefore given as

$$(G_401)$$
 $W = \omega(\mathbf{r},t) \cdot F(\varphi(\mathbf{r},t))$

On a migrant $\varphi = \varphi_0$ the field size *W* of the wave is generally **not** constant.

For $\mathbf{r} = \mathbf{r}_0$ results in the following excitation history:

$$(G_{403}) \qquad W_0 = \boldsymbol{\omega}_0(\mathbf{r}_0) \cdot F(\boldsymbol{v}_0 t)$$

$F(v_0 t)$ is the excitation form (Erregungsform)

To calculate the migrant fields $\varphi(\mathbf{r}, t)$ and the intensity field $\omega(\mathbf{r}, t)$, we set equation G_401 in G_100, using the following abbreviations:

$$(G_{402}) \qquad \dot{F} = \frac{dF}{d\varphi} \qquad \varphi_a \text{ für } \frac{\partial\varphi}{\partial a} \qquad \omega_{ab} \text{ für } \frac{\partial^2\omega}{\partial a\partial b}$$

The result is:

 In the same way, we obtain an expression for W_{abc} .

All members with the common factor ω will now be summarized. The migrants field *M* is independent of the intensity field *L* (interference principle / Interferenzprinzip):

$$(G_406) \qquad \omega \cdot M + L = 0 \qquad M = 0 \qquad L = 0$$

$$(G_{407}) \qquad M = \sum_{ab} B_{ab} \cdot \left(\dot{F} \cdot \varphi_{ab} + \ddot{F} \cdot \varphi_{a} \cdot \varphi_{b} \right) + \sum_{a} C_{a} \cdot \dot{F} \cdot \varphi_{a} + D \cdot F$$

$$(G_{408}) \qquad L = \sum_{ab} B_{ab} \cdot \left(\dot{F} \cdot \omega_{ab} + \dot{F} \cdot \left(\varphi_{a} \cdot \omega_{b} + \omega_{a} \cdot \varphi_{b} \right) \right) + \sum_{a} C_{a} \cdot F \cdot \omega_{a}$$

This is followed by:

- 1. Converting $G_{407}(A_{abc})$ in a vector analytic function of w and div w.
- 2. Summarizing all the members of time by the symbol "- a (**r**, t)". Here are integrated the members of time $\varphi \cdots \varphi$ etc. and the excitation phase increase.
- 3. In equation G_100 C_x , C_y , C_z are proportional to the gradient of a scalar components , which is called χ . In χ goes among other characteristics of the agent as a function of position.

Then we get:

Scalar migrants equation for all linear homogeneous specified field equations:

$$(G_409) \qquad \ddot{F} \cdot \mathbf{w}^2 - \dot{F} \cdot div \ \mathbf{w} - \dot{F} \cdot (\mathbf{w} \cdot grad \ \chi) - a(\mathbf{r}, t) = 0$$

The conversion of the intensity field equation is associated with the treatment of special field equations for determined physical means , i.e. **wave genera** (**Wellengattungen**).

The non-linearity of equation G_{409} has the consequence that their solutions are generally complex functions of the coordinates and the time. Equation G_{406} becomes:

$$(G_410) \qquad \omega^* \cdot F^*(\varphi^*) = (\omega' - i\omega'') \cdot (F' - iF'')$$

Equation G_401 then becomes equation G_411:

The wave history *W* of the same field size *W*:

$$(G_{411}) \qquad W = \frac{1}{2} \cdot \left(\omega \cdot F(\varphi) + \omega^* \cdot F^*(\varphi^*) \right) = \omega' \cdot F'(\varphi', \varphi'') - \omega'' \cdot F''(\varphi', \varphi'')$$

For the purpose of clarity, we convert this general wave expressed in the shape of a product ψ with an always **positive amplitude** *A* and a **change factor** cos Ψ .

$$(G_412) \qquad W = A \cdot \cos \psi$$

Equation G_411 then becomes equation G_413

The wave history *W* of the same field size *W*:

$$(G_{413}) \qquad W = \sqrt{\omega'^2 + \omega''^2} \cdot \sqrt{F'^2 + F''^2} \cdot \cos\left(\operatorname{arc} tg \frac{F''}{F'} + \operatorname{arc} tg \frac{\omega''}{\omega'}\right)$$

IV. Forms of wave excitation

In a concrete wave problem the excitement form **must be given**.

The simplest form of excitement arises when one sets:

$$(G_414) \qquad F(\varphi) = \varphi = \varphi' + i\varphi''$$

Equation G_413 then will change in:

$$(G_{415}) \qquad W = \sqrt{\omega'^2 + \omega''^2} \cdot \sqrt{\varphi'^2 + \varphi''^2} \cdot \cos\left(\operatorname{arc} tg \frac{\varphi''}{\varphi'} + \operatorname{arc} tg \frac{\omega''}{\omega'}\right)$$

The square excitation form is defined by:

$$(G_416) \qquad F(\varphi) = \varphi^2; \quad \dot{F} = 2\varphi; \quad \ddot{F} = 2; \quad \ddot{F} = 0;$$

G_413 changes by square excitation form (quadratische Erregungsform):

Square excitation form:

$$(G_417) F(\varphi) = \varphi^2 = (\varphi'^2 - \varphi''^2) + 2i\varphi'\varphi'' bzw. F' = \varphi'^2 - \varphi''^2 F'' = 2i\varphi'\varphi''$$
The wave history W of the same field size W is therefore given as:

$$(G_418) W = \sqrt{\omega'^2 + \omega''^2} \cdot \sqrt{\varphi'^2 + \varphi''^2} \cdot \cos\left(arc \ tg \frac{2\varphi'\varphi''}{\varphi'^2 - \varphi''^2} + arc \ tg \frac{\omega''}{\omega'}\right)$$
With the following excitation history:

$$(G_419) W_0 = \left(W_0 = \sqrt{\omega_0'^2 + \omega_0''^2} \cdot \sqrt{v_0'^2 + v_0''^2} \cdot \cos\left(arc \ tg \frac{2v_0'v_0''}{v_0'^2 - v_0''^2} + arc \ tg \frac{\omega_0''}{\omega_0'}\right)\right) \cdot t^2$$

G_409 equation thus becomes a **complex migrants equation**: (G_500) $\mathbf{w}^2 - \varphi \cdot div \ \mathbf{w} - \varphi \cdot (\mathbf{w} \cdot grad \ \chi) - a(\mathbf{r}, t) = 0$

By inserting the complex shapes for $\boldsymbol{\phi}$ and its derivatives and separation of real and imaginary parts according to

- $(G_501) \qquad M' + iM'' = 0$
- $(G_502) \qquad L' + iL'' = 0$

$$(G_503) \qquad M'(\varphi',\varphi'') = 0$$

$$(G_504) \qquad M''(\varphi',\varphi'') = 0$$

$$(G_{505}) \qquad L'(\omega', \omega'', \varphi', \varphi'') = 0$$

$$(G_506) \qquad L''(\omega',\omega'',\varphi',\varphi'') = 0$$

the following two real migrants equation are given:

$$(G_{507}) \qquad \mathbf{w}^{\prime 2} - \mathbf{w}^{\prime \prime 2} - \varphi^{\prime} \cdot div \ \mathbf{w}^{\prime} + \varphi^{\prime \prime} \cdot div \ \mathbf{w}^{\prime \prime} + \varphi^{\prime} \cdot (\mathbf{w}^{\prime} \cdot grad \ \chi) - \varphi^{\prime \prime} \cdot (\mathbf{w}^{\prime \prime} \cdot grad \ \chi) - a^{\prime}(\mathbf{r}, t) = 0$$

$$(G_{508}) \qquad 2 \cdot \mathbf{w}^{\prime} \cdot \mathbf{w}^{\prime \prime} - \varphi^{\prime} \cdot div \ \mathbf{w}^{\prime \prime} - \varphi^{\prime \prime} \cdot div \ \mathbf{w}^{\prime} + \varphi^{\prime \prime} \cdot (\mathbf{w}^{\prime} \cdot grad \ \chi) + \varphi^{\prime} \cdot (\mathbf{w}^{\prime \prime} \cdot grad \ \chi) - a^{\prime \prime}(\mathbf{r}, t) = 0$$

And as complex intensity field equation:

$$(G_{509}) \qquad \sum_{ab} B_{ab} \cdot \left(\varphi^2 \cdot \omega_{ab} + 2\varphi \cdot \left(\varphi_a \cdot \omega_b + \omega_a \cdot \varphi_b\right)\right) + \sum_a C_a \cdot \varphi^2 \cdot \omega_a = 0$$

Substituting the complex shapes of φ and its derivatives and separating real and imaginary parts of equation G_416 arise so the following two real intensity field equations:

$$\begin{aligned} \left(\mathbf{G}_{510} \right) & \sum_{ab} B_{ab} \cdot \left[\left(\varphi'^2 - \varphi''^2 \right) \cdot \omega'_{ab} - 2\varphi' \varphi'' \omega''_{ab} + 2 \left(\varphi' \cdot \left(\varphi'_a \omega'_b - \varphi''_a \omega''_b + \omega'_a \varphi'_b \right) - \varphi'' \cdot \left(\varphi'_a \omega''_b + \omega'_b \varphi''_a + \omega'_a \varphi''_b + \omega''_a \varphi''_b \right) \right) \right] + \\ & + \sum_{a} C_a \cdot \left(\left(\varphi'^2 - \varphi''^2 \right) \cdot \omega'_a - 2\varphi' \varphi'' \omega''_a \right) = 0 \\ \left(\mathbf{G}_{511} \right) & \sum_{ab} B_{ab} \cdot \left[\left(\varphi'^2 - \varphi''^2 \right) \cdot \omega'_{ab} + 2\varphi' \varphi'' \omega'_{ab} + 2 \left(\varphi'' \cdot \left(\varphi'_a \omega'_b - \varphi''_a \omega''_b + \omega'_a \varphi''_b + \omega''_a \varphi''_b + \omega'''_a \varphi''_b + \omega''_a \varphi''_b + \omega''_a \varphi''_b + \omega'''_a \varphi''_b + \omega'''_a \varphi''_b + \omega''_a \varphi''_b + \omega'''_a \varphi''_b + \omega'''_a \varphi'''_b + \omega'''_a + \omega'''_a \varphi''_b + \omega'''_a + \omega'''_b + \omega'''_b + \omega'''_$$

The wave equation delivers G_418:

$$(G_{418}) \qquad W = \sqrt{\omega'^2 + \omega''^2} \cdot \sqrt{\varphi'^2 + \varphi''^2} \cdot \cos\left(\operatorname{arc} tg \frac{2\varphi'\varphi''}{\varphi'^2 - \varphi''^2} + \operatorname{arc} tg \frac{\omega''}{\omega'} \right)$$

V. Linear- and Nonlinear waves

If several linear waves are available we obtain G_600 using equation G_401:

$$(G_{600}) \qquad W = W_1 + W_2 + W_3 \dots + W_n = \omega_1 \cdot F_1(\varphi_1) + \omega_2 \cdot F_2(\varphi_2) + \omega_3 \cdot F_3(\varphi_3) + \omega_n \cdot F_n(\varphi_n)$$

We set this equation in G_100. Then all members with the common factor ω will be summarized:

(G_601)
$$\omega_1 \cdot M_1 + \omega_2 \cdot M_2 + ... + \omega_n \cdot M_n + L_1 + L_2 + ... + L_n = 0$$

Linear waves behave as if they were present **alone**. The superimposition is carried out **without interfering** with the propagation of the individual linear waves.

Migrant field and intensity field do not affect each other.

$$(G_{602})$$
 $M_1 = 0; M_2 = 0; \dots M_n = 0$

$$(G_{603})$$
 $L_1 = 0; L_2 = 0; ..., L_n = 0$

If a specified field equation of an agent is **not** linear with respect to one or more partial derivatives of W, we have **nonlinear waves**. When the intensity field of nonlinear waves interfering, they **influence each other**.

VI. Movement

Both migrants φ' and φ'' migrate (move) generally at different speeds:

$$(G_700) \qquad (\mathbf{c'} \cdot \mathbf{n'}) \cdot \mathbf{n'} = \frac{\dot{\boldsymbol{\phi}}}{\mathbf{w'}^2} \cdot \mathbf{w'} \quad ; \quad (\mathbf{c''} \cdot \mathbf{n''}) \cdot \mathbf{n''} = \frac{\dot{\boldsymbol{\phi}}}{\mathbf{w''}^2} \cdot \mathbf{w''}$$

w' and **w**'' and **c**' and **c**' can have different directions of movement. They usually form a splay angle. If the angle is 0° or 180° , so the migrants of the wave are parallel (migrantenparallel), in the other case, they are spread (gespreizt).

In a relative movement between the wave source, the agent and the location of the observer, there are frequency and attenuation differences (Doppler effect), there is the existence of an apparent source (Scheinquelle / Fresnel effect) and beam direction differences (Bradley effect).

VII. Thermal waves

The heat conduction is represented by the following equation:

$$(G_{001}) \qquad \mathbf{W} = -\lambda \cdot grad \ T$$

The temperature propagation in a "homotrop" agent was represented by the following specified field equation:

$$(G_{002}) \qquad D \cdot c \cdot \frac{\partial T}{\partial t} = -div \mathbf{W}$$

For the field variables in the equations G_001 and G_002 must be used:

$$(G_{800}) \qquad T = \boldsymbol{\omega} \cdot F(\boldsymbol{\varphi})$$

The Scalar migrants equation G_409 then gives:

A migrant equation:

$$\begin{pmatrix} G_{801} \end{pmatrix} \quad \ddot{F} \cdot \mathbf{w}^{2} - \dot{F} \cdot div \ \mathbf{w} - \dot{F} \cdot \left(\mathbf{w} \cdot grad \ \log \lambda \right) - \frac{D \cdot c}{\lambda} \cdot \dot{F} \cdot \dot{\phi} = 0$$
An equation for the intensity field:

$$\begin{pmatrix} G_{802} \end{pmatrix} \quad F \cdot div \ grad \ \omega - 2 \cdot \dot{F} \cdot \left(grad \ \omega \cdot \mathbf{w} \right) + F \cdot \left(grad \ \log \lambda \cdot grad \ \omega \right) - \frac{D \cdot c}{\lambda} \cdot F \cdot \dot{\omega} = 0$$

The result of a square excitation according to equation G_416:

A migrant equation:

$$(G_{810}) \qquad \mathbf{w}^2 - \boldsymbol{\varphi} \cdot div \ \mathbf{w} - \boldsymbol{\varphi} \cdot \left(\mathbf{w} \cdot grad \ \log \lambda\right) - 2 \cdot \frac{D \cdot c}{\lambda} \cdot \boldsymbol{\varphi} \cdot \dot{\boldsymbol{\varphi}} = 0$$

An equation for the intensity field:

$$(G_{811}) \qquad \varphi^2 \cdot div \ grad \ \omega - 4\varphi \cdot (grad \ \omega \cdot \mathbf{w}) + \varphi^2 \cdot (grad \log \lambda \cdot grad \ \omega) - \frac{D \cdot c}{\lambda} \cdot \varphi^2 \cdot \dot{\omega} = 0$$

Now we substitute the complex variables. Then we have **two** migrant equations and **two** equations for the intensity field.

Equations G_507 / G_508 becomes G_820 / G_821 Equations G_811 becomes G_830 / G_831

The resut for the **thermal wave** (according to equation G_418) is:

$$(G_{840}) \qquad T = \sqrt{\omega'^2 + \omega''^2} \cdot \sqrt{\varphi'^2 + \varphi''^2} \cdot \cos\left(\operatorname{arc} tg \frac{2\varphi'\varphi''}{\varphi'^2 - \varphi''^2} + \operatorname{arc} tg \frac{\omega''}{\omega'}\right)$$

In this wave equation the following two migrant equations ($G_{820/821}$) and the two equations for the intensity field ($G_{830/831}$) must be set in.

Two migrant equations:

$$(G_{820}) \qquad \mathbf{w}'^{2} - \mathbf{w}''^{2} - \varphi' \cdot div \ \mathbf{w}' + \varphi'' \cdot (\mathbf{w}' \cdot grad \log \lambda) - \varphi'' \cdot (\mathbf{w}'' \cdot grad \log \lambda) - 2 \frac{D \cdot c}{\lambda} \cdot (\varphi' \dot{\varphi}' - \varphi'' \dot{\varphi}'') = 0$$

$$(G_{821}) \qquad 2 \cdot \mathbf{w}' \cdot \mathbf{w}'' - \varphi' \cdot div \ \mathbf{w}'' - \varphi'' \cdot div \ \mathbf{w}' + \varphi'' \cdot (\mathbf{w}' \cdot grad \log \lambda) + \varphi' \cdot (\mathbf{w}'' \cdot grad \log \lambda) - 2 \frac{D \cdot c}{\lambda} \cdot (\varphi'' \dot{\varphi}' - \varphi' \dot{\varphi}'') = 0$$

And:

Two equations for the intensity field:

$$\begin{aligned} (G_{830}) & (\varphi'^{2} - \varphi''^{2}) \Delta \ \omega' - 2\varphi'\varphi'' \ \Delta \ \omega'' - 4\varphi' \cdot (\nabla \ \omega' \cdot \mathbf{w}') \cdot (\nabla \ \omega' \cdot \mathbf{w}' - \nabla \ \omega''' \cdot \mathbf{w}'') + \\ & + 4\varphi'' \cdot (\nabla \ \omega'' \cdot \mathbf{w}' + \nabla \ \omega' \cdot \mathbf{w}'') + (\varphi'^{2} - \varphi''^{2}) (\nabla \ \log \ \lambda \cdot \nabla \ \omega') - \\ & - 2\varphi'\varphi'' \cdot (\nabla \ \log \ \lambda \cdot \nabla \ \omega'') - \frac{D \cdot c}{\lambda} \cdot (\varphi'\dot{\omega}' - \varphi''\dot{\omega}'') = 0 \end{aligned}$$

$$(G_{831}) & \varphi'\varphi'' \ \Delta \ \omega' + 2(\varphi'^{2} - \varphi''^{2}) \ \Delta \ \omega'' - 4\varphi'' \cdot (\nabla \ \omega' \cdot \mathbf{w}' - \nabla \ \omega'' \cdot \mathbf{w}'') - \\ & - 4\varphi' \cdot (\nabla \ \omega'' \cdot \mathbf{w}' + \nabla \ \omega' \cdot \mathbf{w}'') + 2\varphi'\varphi'' \cdot (\nabla \ \log \ \lambda \cdot \nabla \ \omega') + \\ & + (\varphi'^{2} - \varphi''^{2}) \cdot (\nabla \ \log \ \lambda \cdot \nabla \ \omega'') - \frac{D \cdot c}{\lambda} \cdot (\varphi'\dot{\omega}'' - \varphi''\dot{\omega}') = 0 \end{aligned}$$

VIII. Literature

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